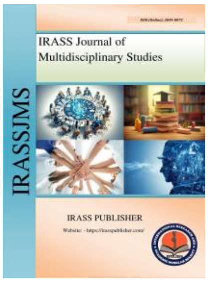
IRASS Journal of Multidisciplinary Studies
Abbriviate Title- IRASS J Mul Stud
ISSN *(Online) 3049-0073*
https://irasspublisher.com/journal-details/IRASSJMS
Vol-1, Iss-3 (December-2024)



Thermal Explosion with Arrhenius Kinetics in Parallel Plates Subject to Various Boundary Conditions

Dr. Akinbobola, Akinyemi Olufunminiyi[1], Moses, Felix Oghenerobor[2], Dr. AKINOLA Emmanuel Taiwo[3*], OBAMOYEGUN Oluwaponmile Joseph[4]

[*1] Department of Curriculum and Instruction, Adeyemi Federal University of Education, Ondo, Ondo State, Nigeria

[2] Department of Mathematics, Adeyemi Federal University of Education, Ondo, Ondo State, Nigeria

[*3] PhD in Business Administration (Entrepreneurship and Human Resource Management), Statistics and Records, Adeyemi Federal University of Education, Ondo, Ondo State, Nigeria

[4] Ph.D Candidate in Accounting, Directorate of Audit, Adeyemi Federal University of Education, Ondo, Ondo State, Nigeria

Corresponding Author **Dr. AKINOLA Emmanuel Taiwo**

PhD in Business Administration (Entrepreneurship and Human Resource Management), Statistics and Records, Adeyemi Federal University of Education, Ondo, Ondo State, Nigeria

Article History

Received: 15 / 12 / 2024

Accepted: 28 / 12 / 2024

Published: 31 / 12 / 2024

Abstract: The study considers a combination of symmetric and asymmetric boundary conditions on thermal explosion with Arrhenius kinetics for a slab. Realistic assumptions will convert the energy conservation equation into dimensionless form, and the resulting nonlinear ordinary differential equation under various boundary conditions will be analytically solved using Frank-Kamanetskii's methods. This analytical method will yield the critical ignition parameters and the critical ignition temperature. The results were compared with previous studies in literature.

Keywords: Arrhenius Kinetics, Thermal Explosion, Frank-Kamenetskii number, Symmetric Boundary Condition, Asymmetric Boundary Condition.

Introduction

It was demonstrated that the original formulation of the problem by Semenov (1928) correctly characterizes the phenomena and explains the cause of the explosion after a review and analysis of the historical evolution of the thermal-explosion theory. Semenov demonstrated that an explosion happens when a solid's internal heat creation surpasses its external heat dissipation. According to Frank-Kamenetskii's (1969) critique of Semenov's reasoning, the explosion was caused by a temperature differential between the solid's surface and center. But his well-known and clever small-temperature model and the resulting differential equation solution warped the issue and slowed down the process of fully comprehending it.

Frank-Kamenetskii came to the conclusion that an explosion happens when there is no way to solve the issue. The validity of Semenov's formulation was confirmed by Donaldson and Tsao's (2006) precise solution to the problem. The problem's physics was fully understood after more research on how reactant use affected the issue.

Thermal Explosion may be due to a violent reaction between the overcharged anode and the high temperatures of the electrolytes that results in an exothermic reaction between the delithiated cathode and electrolyte by Ohsaki (2005).

When a chemical system goes through an exothermic reaction, insufficient heat is extracted from the system, causing the reaction process to become self-heating. This results in a thermal explosion.

However, thermal explosion theory is based on the idea that progressive heating raises the rate at which heat is released by the reaction until it exceeds the rate of heat loss from the area. At a given composition of the mixture and a given pressure, explosion will occur at a specific ignition temperature that can be determined from the calculation of heat loss and heat gain.

The transition from Combustion to Explosion is caused by an acceleration of the reaction induced either by a raise in temperature or by increasing length of the reaction Chain. The first is called thermal explosion, and the second is called Chain explosion.

If the heat released by the exothermic chemical reaction exceeds the rate at which heat is lost from the body to its surroundings, then an unstable build-up of heat inside a body can occur. This is because the increase in the internal temperature causes the reaction rate to increase, so more heat is generated, leading to a further increase in temperature. If the increase in the reaction rate outweighs the increase in the rate at which heat is lost to the surroundings, an unstable thermal runaway /explosion can occur (Brain, 2019).

Objectives of the Study

The specific objectives of the study are to;

i. Formulate an energy balance equation for the thermal explosion with Arrhenius kinetics in parallel plates;
ii. Solve the formulated energy equation using standard techniques; and
iii. Examine the effects of various boundary conditions on the thermal explosion with Arrhenius kinetics.

Conceptualization/Definition of Terms

For a better understanding of later discussions, the following definitions of important terms used in this study are given below.

Definition 1.1: (Combustion)

Combustion: is a chemical process or a reaction between fuel and oxygen which releases heat and light energy.

General Equation:

$$Fuel + O_{2(g)} \rightarrow Waste + Energy$$

Definition 1.2: (Chemical Kinetics)

Chemical Kinetics: is the study of the rates of chemical reactions.

Definition 1.3: (Steady state)

A system is said to be steady when variables (such as velocity, temperature, density etc.)

Constant with time (that is $\frac{\partial}{\partial x} = 0$) (Schlichting & Gersten, 2017).

Definition 1.4: (Arrhenius Equation)

An Arrhenius Equation: gives the dependence of the rate constant of a chemical reaction on

the absolute temperature, a pre-exponential factor and other constants of the reaction.

Definition 1.5: (Boundary Conditions)

Boundary conditions: this condition specifies the value that a solution must take in some

region of space and it is independent of time.

Definition 1.6: (Thermal Explosion)

Thermal Explosion: An explosion is a rapid increase in volume and release of energy in an

extreme manner, usually with the generation of high temperatures and the release of gases. It's

an event that once initiated grows rapidly and initially unbounded.

Definition 1.8: (Self-Heating)

Self-Heating: is a process where a material increases in temperature due to the release of heat

from ongoing chemical reactions and without drawing heat from its surroundings.

Definition 1.9: (Criticality)

The critical point of a function of a single real variable, f(x) is a value in the domain of f

where its derivative is 0 (i.e. $f^{'}(x) = 0$).

This is the conversion of one form of energy (such as nuclear, electrical, chemical energy) into

heat energy in a medium. A lot of heat transfer applications involve heat generation and this heat

generation according to (Azim and Chowdhury, 2013) modifies the temperature distribution.

Heat transfer by natural convection significantly influences temperatures of power generating systems (Theodore et al., 2011).

Statement of Problem

The study of thermal explosion with Arrhenius kinetics has attracted a great deal of interest

from researchers in the past few decades because of its occurrence, as seen in volcanic eruption and nuclear plants. However, relatively little is known about the combined mixture of symmetric and asymmetric conditions on thermal explosion with Arrhenius kinetics for a slab; hence, this study.

Methodology

The study presents the fundamental energy equation for the thermal explosion with Arrhenius kinetics in parallel plates under varied boundary conditions. First, the fundamental equation of energy is presented, followed by a discussion of the model assumptions that were used to create the governing equations. The dimensionless Heat transport properties are covered and variables utilized in non-dimensionalizing the constructed equation are defined. A demonstration and discussion of the integration technique used to solve the given problem are provided.

Basic Equations

Equation of Conservation of Energy

The law of conservation of energy states that the amount of work done on a system and the amount of heat it receives cause the system's internal energy to rise proportionately. The energy equation controls heat transport in a medium when chemical reactions and insignificant radiant energy are present;

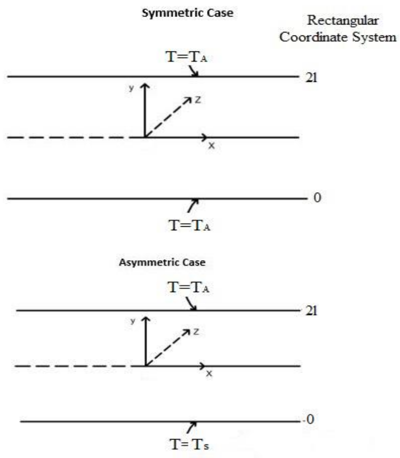
$$\rho Cp \frac{DT}{Dt} = K\nabla^2 T + \phi + e_g \qquad (1)$$

Where the specific heat at constant pressure, T is the temperature, is the viscous dissipation function, k is the thermal conductivity, is the heat generation term.

Problem Formulation

A stationary one-dimensional parallel plate of a reacting material having thickness 2L with its plane surfaces maintained at different boundary conditions is being considered.

Figures 1 and 2 show the geometry and co-ordinates system of our problem.



Results and Discussion

The energy equation in (3.1) above was reduced with some assumptions to give

$$k\frac{d^2T}{dx^2}+A\Delta R\rho\exp(-E/RT)=0 \quad (3.2)$$

An equation (3.2) is the energy equation subject to some assumptions used.

The temperature T and distance y are rescaled so as to compare terms in the equations without reference to units. The temperature equations are non-dimensionalized with the variables;

$$\begin{cases} z=\dfrac{x}{l} \\ \theta=\dfrac{E}{RT_A}(T-T_A)=\dfrac{1}{\beta T_A}(T-T_A) \end{cases} \quad (3.4)$$

The equation (3.2) is reduced to give a dimensionless temperature equation

$$\frac{d^2\theta}{dz^2}=-A\Delta H_{R\rho}\left(\frac{r^2}{k}\right)\left(\frac{E}{RT_A{}^2}\right)\exp\left(-\frac{E}{RT_A}\right)\exp\left(\frac{\theta}{1+\beta\theta}\right) \quad (3.5)$$

Where

$$\delta=A\Delta H_{R\rho}\left(\frac{r^2}{k}\right)\left(\frac{E}{RT_A{}^2}\right)\exp\left(-\frac{E}{RT_A}\right)\exp\left(\frac{\theta}{1+\beta\theta}\right) \quad (3.6)$$

is the Frank-Kamenetskii number.

$$\frac{d^2\theta}{dz^2}=-\delta\exp\left(\frac{\theta}{1+\beta\theta}\right) \quad (3.7)$$

When $\beta\to 0$, equation r (3.7) reduced becomes;

$$\frac{d^2\theta}{dz^2}=-\delta\exp(\theta) \quad (3.8)$$

The linear ordinary differential equation (3.8) can be solved by multiplying both of its sides by

$$2\frac{d\theta}{dz}$$

$$2\frac{d\theta}{dz}\frac{d^2\theta}{dz^2}+2\delta\exp(\theta)\frac{d\theta}{dz}=0 \quad (3.9)$$

By integrating both sides of Equation (3.9), we have;

$$\left(\frac{d\theta}{dz}\right)^2+2\delta\exp(\theta)=A_0 \quad (4.0)$$

NOTE: A_0 is the constant of integration.

By separating the variables in Equation (4.0) above gives,

$$\frac{d\theta}{\sqrt{A_0-2\delta\exp(\theta)}}=dz \quad (4.1)$$

Integrating both sides, we have;

$$\int\frac{d\theta}{\sqrt{A_0-2\delta\exp(\theta)}}=dz \quad (4.2)$$

Note that the left hand side is integrated by replacing A_0 with $2\delta A$.

$$\int\frac{d\theta}{\sqrt{2\delta A-2\delta\exp(\theta)}}=z+A_1 \quad (4.3)$$

By factoring out $\sqrt{2\delta A}$ and multiplying both sides by $\sqrt{2\delta A}$, we have,

$$\int\frac{d\theta}{\sqrt{1-A^{-1}\exp(0)}}=\pm\sqrt{2\delta A}z+B \quad (4.4)$$

Where B = $\left(\sqrt{2\delta A}\right)A_1$

By letting $\theta=-2\ln P=-2\,dp/p$ L.H.S of Equation (4.4) above becomes;

$$-2\int\frac{dp}{\sqrt{\left(1-A^{-1}P^{-2}\right)}}=-2\int\frac{dp}{p\sqrt{\left(1-A^{-1}P^{-2}\right)}} \quad (4.5)$$

Equation (4.5) becomes,

$$-2\int\frac{dp}{p\sqrt{\left(1-A^{-1}P^{-2}\right)}}=\pm\sqrt{2\delta A}z+B \quad (4.6)$$

Dividing both sides by -2 gives,

$$\int\frac{dp}{\sqrt{\left(P^2-A^{-1}\right)}}=\sqrt{\frac{\delta A}{2}}z-B \quad (4.7)$$

Letting,

$P=\sqrt{A^{-1}}\cosh q$, the L.H.S of the Equation becomes;

$$\int\frac{(\sqrt{A^{-1}}\sinh q)dq}{\sqrt{A^{-1}(\cosh^2 q-1)}}=\int dq=q \quad (4.8)$$

From the above,

$\theta=-2\ln P$ and $\sqrt{A^{-1}}\cosh q\Rightarrow q=\cosh^{-1}(\sqrt{A}\exp(-\theta/2)$

Hence, Equation (4.8) becomes,

$$\sqrt{A}\exp(-\theta/2)=\cosh\left(\sqrt{\frac{\delta A}{2}}z-B\right. \quad (4.9)$$

Divide both sides by $\sqrt{A}$

$$\exp\left(-\theta/2\right)=\cosh\frac{\left(\sqrt{\frac{\delta A}{2}}z-B\right)}{\sqrt{A}} \quad (5.0)$$

Squaring both sides, and taking the reciprocal, we have;

$$\exp(\theta) = \frac{A}{\cosh^2(\sqrt{\frac{\delta A}{2}}z - B} \quad (5.1)$$

Taking ln of both sides, we have;

$$\theta(z) = \ln(\frac{A}{\cosh^2\sqrt{\frac{\delta A}{2}}z - B}) \quad (5.2)$$

Equation (5.2) represents the temperature equation of the parallel plates with Arrhenius kinetics which will be analyzed for four (4) different boundary conditions.

Here, A and B are integration constants, z is the dimensionless distance and δ is the Frank-Kamenetskii parameter.

Case 1: Symmetric Boundary Condition

The symmetric boundary condition given as;

$$\begin{cases} T = T_A \text{ at } x = 0 \\ T = T_A \text{ at } x = 2l \end{cases} \quad (5.3)$$

is non-dimensionalized with the variables in Equation (3.4) to give,

$$\begin{cases} \theta = 0 \text{ at } z = 0 \\ \theta = 0 \text{ at } z = 2 \end{cases} \quad (5.4)$$

Substituting the boundary condition $\theta(0) = 0$ into Equation (5.2) above gives,

$$0 = \ln\left[\frac{A}{\cosh^2(-B)}\right] \quad (5.5)$$

Equation (5.5) reduced to;

$$A = \cosh^2(-B)$$

$$\Rightarrow A = \cosh^2 B$$

(5.6)

By substituting the boundary condition $\theta(2) = 0$ into Equation (5.2) above gives,

$$0 = \ln\left[\frac{\cosh^2 B}{\cosh^2\sqrt{2\delta}\cos B - B}\right] \sqrt{2\delta} = \frac{2B}{\cosh B} \quad (5.7)$$

Taking exponential of both sides gives,

$$\cosh^2 B = \cosh^2(\sqrt{2\delta}\cosh B - B) \quad (5.8)$$

$$B = \sqrt{2\delta}\cosh B - B \quad (5.9)$$

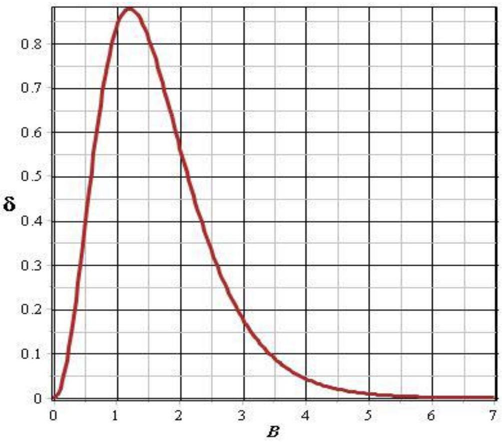
$$\sqrt{2\delta} = \frac{2B}{\cosh B} \quad (6.0)$$

Squaring both sides and dividing by 2

$$\delta = \frac{2B^2}{\cosh^2 B} \quad (6.1)$$

Since $\delta = \delta(B), \delta$ is plotted against B to obtain δ_{cr} and the value of B (6.2)

A graph of δ against B for symmetric boundary condition.



From the graph above $\delta_{cr} = 0.878$ and the value of $B = 1.193$ from equation (5.6), $A = \cosh^2(1.193)$ =3.240 (6.3)

Substituting the values of A, B A, B and δ_{cr} into equation (5.2) gives,

$$\theta(Z) = \ln\left(\frac{3.240}{\cosh^2(\frac{\sqrt{(0.878)3.240}}{2}z - 1.193}\right) \quad (6.4)$$

$$\theta(Z) = \ln\left(\frac{3.240}{\cosh^2(1.193z - 1.193}\right) \quad (6.5)$$

The graph of $\theta_{\max}$ against z is plotted and discussed.

Case 2: Asymmetric Boundary Condition

The asymmetric boundary condition

$$\begin{cases} T = T_A \text{ at } X = 0 \\ T = T_S \text{ at } X = 2l \end{cases} \quad (6.6)$$

is non-dimensionalized with the variables in Equation (3.4) to give

$$\begin{cases} \theta = 0 \text{ at } z = 0 \\ \theta = \theta_S \text{ at } z = 2 \end{cases} \quad (6.7)$$

Substituting the boundary condition $\theta(0) = 0$ into equation (5.2) gives,

$$0 = \ln\left[\frac{A}{\cosh^2(-B)}\right] \quad (6.8)$$

$$\Rightarrow A = \cosh^2 B \quad (6.9)$$

By substituting the boundary condition $\theta(2) = \theta_S$ into equation (5.2) gives,

$$\theta_S = \ln\left[\frac{\cosh^2 B}{\cosh^2(\sqrt{2\delta}\cosh B - B)}\right] \quad (7.0)$$

By taking exp of both sides gives,

$$\text{Exp}(\theta_S) = \frac{\cosh^2 B}{\cosh^2(\sqrt{2\delta}\cosh B - B)} \quad (7.1)$$

Again, taking square root of both sides,

$$\pm\sqrt{\exp(\theta_S)} = \frac{\cosh B}{\cosh(\sqrt{2\delta}\cosh B - B)} \quad (7.2)$$

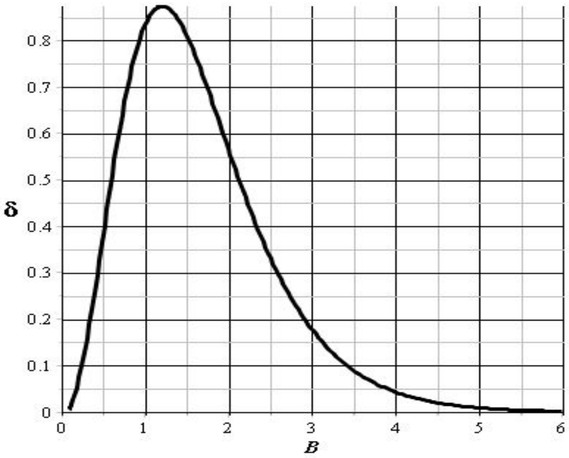
$$\cosh(\sqrt{2\delta}\cosh B - B) = \pm\exp\left(-\theta_S/2\right)\cosh B \quad (7.3)$$

$$\sqrt{2\delta}\cosh B = B + \cosh^{-1}\{\pm\exp(-\theta_S/2)\cosh B\}$$
(7.4)

Squaring both sides and dividing through by $2\cosh^2 B$

$$\delta = \frac{1}{2}\left[\frac{B+\cosh^{-1}\{\pm\exp(-\theta_S/2)\cosh B\}}{\cosh B}\right]^2 \quad (7.5)$$

Here, δ_{cr} is obtained by plotting δ against B, while the surface is assumed to be at temperature $\theta_S = 0.01$.



Plot of δ against B for asymmetric boundary condition.

From the graph above $\delta_{cr} = 0.874$ and the value of $B = 1.197$

From equation (6.9),

$$A = \cosh^2(1.197) = 3.262 \quad (7.6)$$

Substituting A, B and δ_{cr} into equation (5.2) gives,

$$\theta(Z) = \ln\left(\frac{3.262}{\cosh^2(\sqrt{\frac{(0.874)3.262}{2}} - 1.197}\right) \quad (7.7)$$

$$\theta(Z) = \ln\left(\frac{3.262}{\cosh^2(1.194Z - 1.197)}\right) \quad (7.8)$$

The graph of $\theta_{\max}$ and Z is plotted, and therefore discussed below.

Results Discussion and Findings

In this section, effect of the various boundary conditions are presented graphically and their effects on the temperature profiles are discussed. Except in cases where they are indicated as variable parameters, the following constant values are assumed for the relevant parameters: and.

Figure 4.1 represents the plots of δ against B and θ against z. The plots in figure 4.1a, shows that moving along B, the Frank Kamenetskii parameter δ, start increasing from zero to the maximum value of $\delta = 0.878$ at $B = 1.193$. The maximum point $\delta = 0.878$ beyond which the curve of θ starts decreasing is the critical Frank Kamenetskii δ_{cr}. Beyond δ_{cr}, the curve decreases monotonically to zero and maintains zero value for $B > 5.5$.

Figure 4.1b; depict a parabolic curve bounded above for the plot of $\theta_{\max}$ against the dimensionless distance z.

It is seen that, $\theta_{\max}$ increases uniformly to a peak value of 1.176. Also, along z, $\theta_{\max}$ increases uniformly for to a zero for Z > 1

Figure 4.1 therefore reveal that a parallel plate of dimensionless thickness of 2 with Arrhenius Kinetic will have a critical Frank Kamenetskii value of $\delta_{cr} = 0.878$ and a critical maximum temperature of $\theta_{\max cr} = 1.176$ when the walls are subjected to symmetric dimensionless temperature $(\theta = 0)$.

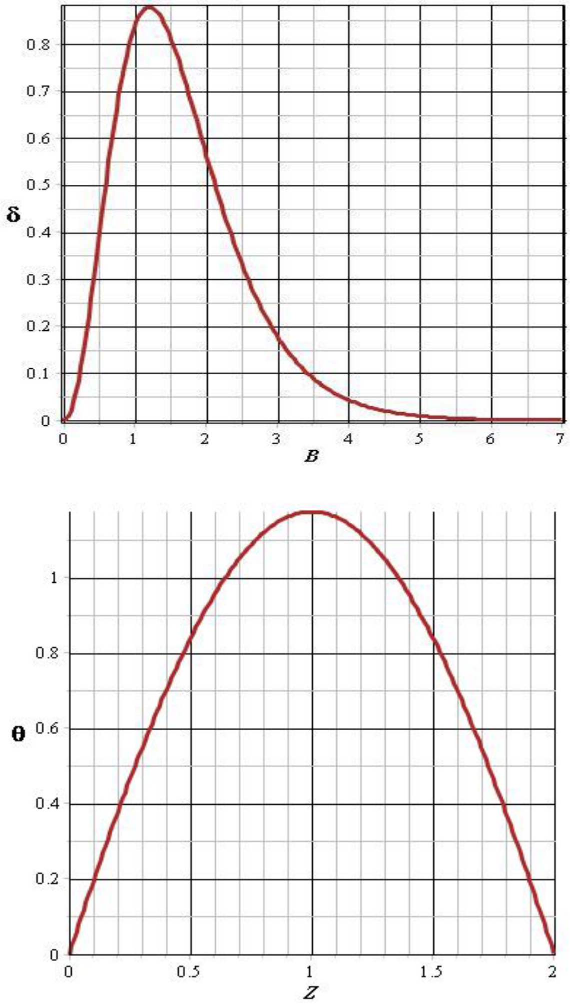
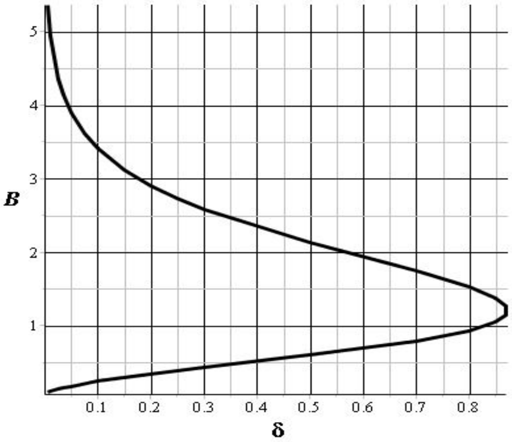


Figure 4.1: Graphs of (a) δ against B and (b) θ against Z for symmetric boundary condition.

However, figure 4.2 represents the graphs of δ and B and θ and z. The plots in figures 4.2a, shows that, moving along B, the Frank Kamenetskii parameter δ increases from zero to a maximum value of $\delta = 0.874$ at the point where B =1.197. The maximum point $\delta = 0.874$ where the value of θ starts decreasing is the critical Frank Kamenetskii δ_{cr} . Beyond δ_{cr} the curve decreases monotonically to zero and maintain zero value for $B > 5.5$

Figures 4.2b, shows a parabolic curve of $\theta_{\max}$ against the dimensionless distance z. it is seen that, $\theta_{\max}$ increases uniformly to a peak value of 1.176. Also, along z, $\theta_{\max}$ increases uniformly for z <1 and decreases uniformly to zero for z >1

Figure 4.2 therefore reveals that the parallel plates will have a critical Frank Kamenetskii value of $\delta_{cr} = 0.874$ and a critical maximum temperature ($\theta = 0)$ at z = 0 and $(\theta = \theta_S = 0.01)$ at z =2.



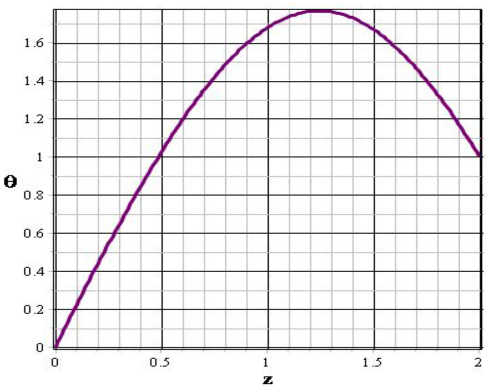


Figure 4.2: plots (a) δ against B and (b) θ against z for asymmetric boundary condition

Conclusion

From this paper, the following significant conclusions were drawn:

i. The maximum critical temperature $\theta_{\max cr}$ increases with increasing θ_S and decreasing δ_{cr} for the asymmetric boundary condition model.

ii. The symmetric boundary condition model serves a limiting case to the asymmetric boundary condition model.

iii. The maximum critical temperature $\theta_{\max cr}$ and the critical Frank-Kamenetskii parameter δ_{cr} of the parallel plates changes significantly as the boundary conditions changes.

Recommendation

Based on the findings, it is therefore recommended that, the right and accurate information about the maximum critical temperature $\theta_{\max cr}$ and the critical Frank-Kamenetskii parameter δ_{cr} of materials should be known and taken into consideration in industries and working environments. This is to ensure safety of both workers and the environment.

Acknowledgments:

Services involved: The authors gratefully acknowledge the support of the Directorate of Academic Planning, Department of Curriculum and Instruction, Department of Mathematics, Division of Statistics and Records, Directorate of Audit of Adeyemi Federal University of Education, Ondo, Nigeria for the support and authorization obtained to conduct this study, as well as for the helpful guidance provided during the investigation. We are grateful to the Management of Adeyemi Federal University of Education in Ondo, Nigeria, for their hard work in supporting our research and providing insightful guidance.

Funding: The Tertiary Education Trust Fund (TetFund), which provided financing for the research through Institution-Based Research (IBR), is acknowledged by the authors.

References

1. Adler J., & Enig, J. W. (1964). *The critical conditions and thermal explosion theory with reactant consumption.* University of London 8, 97-103.

2. Adler, J. (1991). *Thermal explosion theory with Arrhenius kinetics. Homogeneous and inhomogeneous media*. Proceedings of the royal society of London A 329-335.

3. Adler, J., & Enig, J. W. (2013). Explosion characteristics of a volatile explosive. *The open Thermodynamics journal*, 7, 88-102.

4. Adegbie, K. S., & Alao F. I. (2006). *On thermal explosion of sensitized reaction with variable heat loss.* Department of mathematical sciences, federal university of technology, Akure, Nigeria.

5. Ajadi, S.O., & Nave, O. (2010). Approximate critical conditions in thermal explosion theory for a two-step kinetic model. *Journal of Mathematical Chemistry*, DOI: 10.1007/s10910-009-9600-y.

6. Ajadi, S.O. & Nave, V. (2009). Critical behaviour in a three step reactions kinetics model. *Combustion Theory and Modeling*. 13 (1):, 1–16.

7. Amos, O. P. (2011): The effect of activation energies on thermal explosion in the interior of the earth. *Journal of Mathematics and Statistics, Science Publications,* New York, USA 7 (3), 222-226.

8. Arvind, V., Massimo, M., & Hua, W. (2010). *Parametric sensitivity in chemical Systems.* Cambridge University Press, New York: 10th edition, 36-79.

9. Ayeni, R.O., Akinpelu, F. O., & Popoola, A. O. (2007a). Effect of radioactive heat sources and gravitational differentiation on thermal explosions in the early evolution of the Earth. *Science Focus12* (1), 136-138.

10. Ayeni, R.O., Akinpelu, F. O., & Popoola, A. O. (2007b). Some remarks on thermal explosions in the early evolution of the Earth. *Science Focus*, 12 (1), 1-3.47.

11. Ayeni, R.O., Akinpelu, F. O., & Popoola, A. O. (2008). Numerical solution of the energy equation associated with early evolution of the Earth. *Science Focus,* 13 (2), 70-72.

12. Ayeni, R.O., Popoola A. O. & Fenuga, O.J., (2006). Remarks on thermal explosions in the early evolution of the Earth. *Journal of Nigerian Association of Mathematical Physics*, 10: 069-070.

13. Brain, G. H. (2019). An Introduction to the theory of thermal explosion https://www.researchgate.net/publication/333773680.

14. Buckmaster, J. D., & Ludford, G. S. S., (1992). *Theory of laminar flames*. Cambridge University Press Cambridge, 1, 115-118.

15. Camphell, A. (2015). The effect of external heat transfer on thermal explosion in a spherical vessel with natural convection. *Physical Chemistry Chemical Physics,*
16. 26), 16894-16906.
17. Dass, H. K. (2013). *Advanced engineering mathematics*, 21st revised edition, pp. 1119, S. Chand and Company Ltd, New Delhi, India.

18. Filimonov, V. Y., & Koshelev, K. B. (2019). *Critical autoignition conditions for arbitrary reaction order.* Proceeding of the Royal Society A. 475 (2227): 1-15.

19. Frank-Kamenetskii, D. A. (1969). Diffusion and Heat Transfer in Chemical Kinetics 2nd Edition (ed. J. P. Appleton), *Plenum*, 225-226.

20. Gustafson, K. E. & Eaton, B. E. (1982). Exact solutions and ignition parameters in the Arrhenius conduction theory of gaseous thermal explosion. Zeitschriftfur Angewandte Mathematik Und Physics (ZAMP), 33, 392–405.

21. Lebelo, R. S., Moloi, K.C, Chitumwa, C. C. & Adesanya, S. O. (2018). Radiation and thermal decomposition analysis in a reactive slab of variable thermal conductivity. *International Journal of Engineering and Technology,* 7 (3.19), 27-32.

22. Makinde, O. D. & Osalusi, E. (2005). Exothermic explosions in symmetric geometries: an exploitation of perturbation technique. *Romanian Journal of Physics,* 50 (5–6): 621–625.

23. Novozhilov V. (2018). Kinetic effects in thermal explosion with oscillating ambient conditions. *Scientific Reports,* 8 (1), 1-12.

24. Ohsaki, T ,Takashi Kishi, Kuboki T., Takami N., Shimura N., Sato Y., Sekino M. & Satoh, A. (2005). Overcharge reaction of lithium-ion batteries. *Journal of Power Sources,* 146 (1): 97-100.

25. Okoya, S. S. (2004). Reactive-diffusive equation with variable pre-exponential factor. *Mechanics Research Communications*, 31: 263–267.

26. Okoya, S. S. (2018). *The many 'faces' of thermal explosion: Mathematical Modeling Point of View,* 329 Inaugural Lecture, Obafemi Awolowo University, Ile-Ife.

27. Okoya, S. S. (2006). Ignition and transition conditions in the theory of combustion. *IfeJournal of Science,* 8 (2), 51-54.
28. Okoya, S. S. (2013). On criticality and disappearance of criticality for a branched chain thermal reaction with distributed temperature. Afrika Matematika, 24 (4): 1-12.

29. Olanrewaju, P. O., Egunjobi A.S. & Owoloko, A. E. (2014). Thermal explosion analysis of a strong exothermic chemical reaction with variable pre-exponential factor in a spherical vessel. *American Journal of Scientific and Industrial Research, 3269,* 1-4.

30. Pedro, J. V. A. (2016). Chemical Kinetics: Rates of reaction. *Cubel consultancy presentation,* 1, . 3.

31. Popoola, A. O. & Ayeni, R.O. (2005). On the Dependency of activation energies of a two steps Arrhenius reaction. *Journal of Mathematical Association of Nigeria (abacus),* 32 (2a), 79-86.

32. Popoola, A. O. (2007). *Mathematical theory of two-step Arrhenius reactions*. PhD Thesis, Ladoke Akintola University of Technology LAUTECH, Ogbomoso, Nigeria.

33. Prabhakar, D. R., Paul, R. A. & Michael, J. P. (1998). Effect of inserts on layer Ignition temperatures of coal dust. *Combustion and flame,* 114, 41–53.

34. Popoola, A. O. (2014). On thermal explosion arising from time-depend gravitational differentiation and radioactive decay in the earth's intro.
35. *European Scientific* Journal, 10, (21), 1857 – 7881.

36. Schlichting, H. & Gersten, K. (2017). *Boundary layer theory*, 9th Edition edition, pp. 59-62, Springer-Verlag Berlin Heidelberg.

37. Semenov, N. N. (1928). On the theory of combustion processes. *Russian Journal of Physical Chemistry*, 60 (3), 247–250.

38. Shouman, A. R. (2006). A review of one aspect of the thermal-explosion theory. *Journal of Engineering Mathematics*, 56, 179–184.

39. Sulyman, S., Oderinu, R. & Ohaegbue, A. D. (2019). Thermal explosion and irreversibility of hydromagnetic reactive couple stress fluid with viscous dissipation and Navier slips. *Theoretical and Applied Mechanics Letters,* 9 (4), 246-253.

40. Theodore, L. B., Adrienne, S. L., Frank, P. I., David, P. D. (2011). *Fundamentals of heat and mass transfer*, 7th edition, pp. 378, John Wiley and Sons, Inc.

41. Villemur, C., Lucas P., Nathalie, B. & Patricia, R. (2019). Runaway reaction hazard assessment for chemical processes safety, *Chemical Engineering Transactions*, 77: 451-456.

42. Vityazev, A. V. (2004). Thermal explosions in the early evolution of the earth. *Combustion, Explosion and Shock Waves*, 40 (6), 720-723.

43. Zel'dovich, Y. B., Barenblatt, G. I., Librovich, V. B., & Makhviladze, G. M. (1980). Mathematical theory of combustion and explosion, Nauka, Moscow.